

# A Conformal Mapping-based Framework for Robot-to-Robot and Sim-to-Real Transfer Learning

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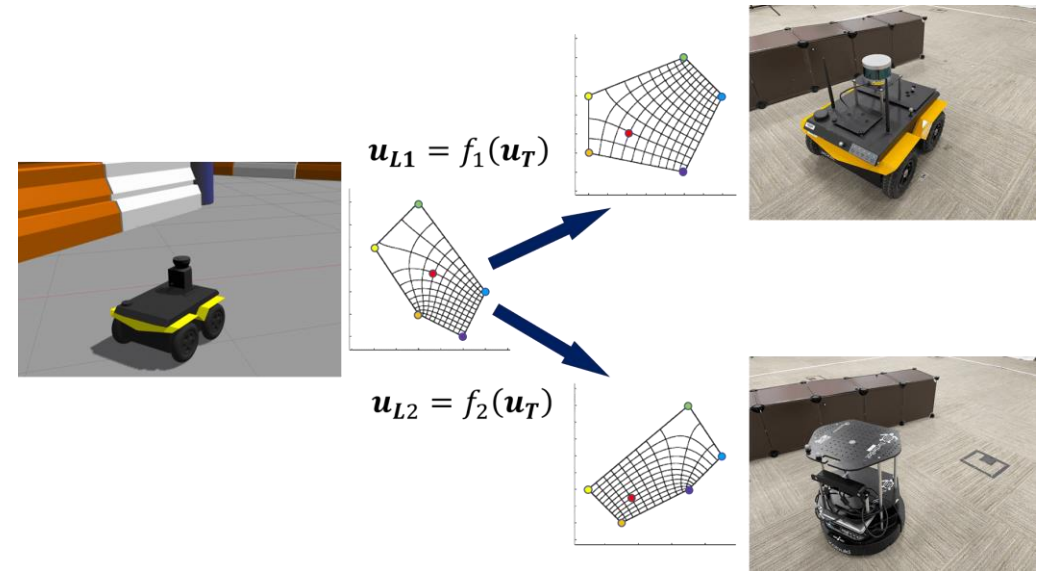
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# Introduction

## Motivations:

- Planning and control methods developed in Simulations can **behave differently** on real systems.
- The algorithm designed specifically for one platform is **hard to be transferred** to other systems.
- The same system needs to be adjusted **over time** or under **different environments**.



## Goals:

- Reduce the sim-to-real gap to **boost** the deployment of **motion planning** and **control methods**.
- **Transfer knowledge** designed for a specific robot onto a different robot.
- Quickly learn the limits and proper input mapping to **compensate** for system **deterioration**.

# Problem Definition

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## 1. Teacher-Learner Control Transfer:

- Given a teacher robot **with dynamics**  $x_T(t + 1) = f_T(x_T(t), u_T(t))$  and **control law**.
- Given a learner robot **without** knowing its **dynamics**  $f_L$ .
- Find a policy  $g$  to map the input from teacher to the learner, such that,

$$u_L(t) = g(u_T(t))$$

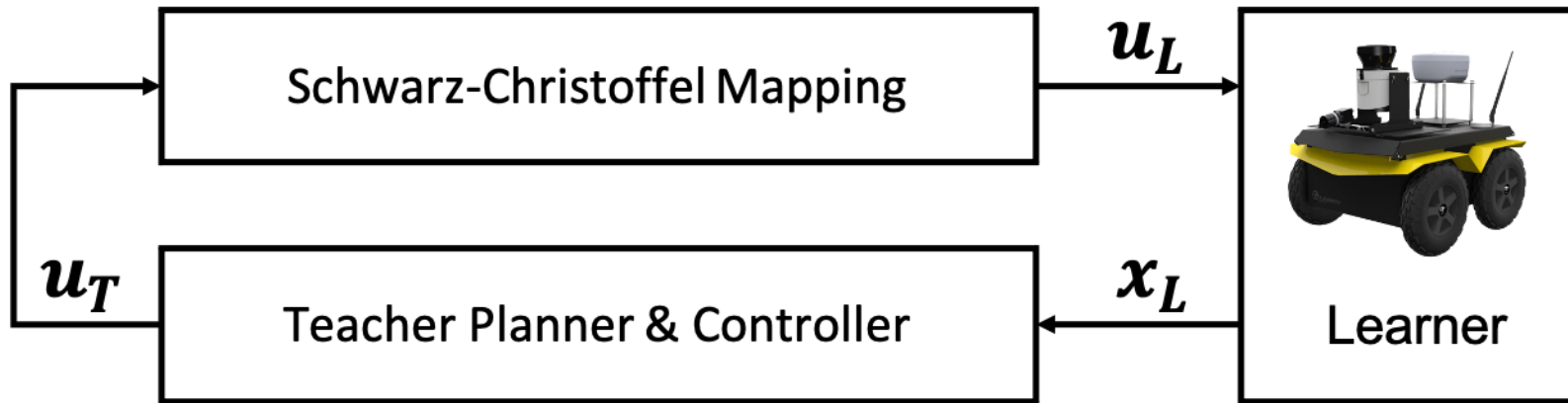
$$f_T(x_T(t), u_T(t)) = f_L(x_L(t), u_L(t))$$

## 2. Teacher-Learner Motion Planning Adaptation

- Assume that the learner has less capability than the teacher
- Design a teacher's planning policy  $\pi_T^L$  such that the computed trajectory **adapts to** the **limitation** of the learner

# Methodology

- **Schwarz-Christoffel mapping (SCM)** based command mapping from the teacher's command domain to the learners.
- **Motion Primitive** based path planning limits the trajectory within the learner's capability.



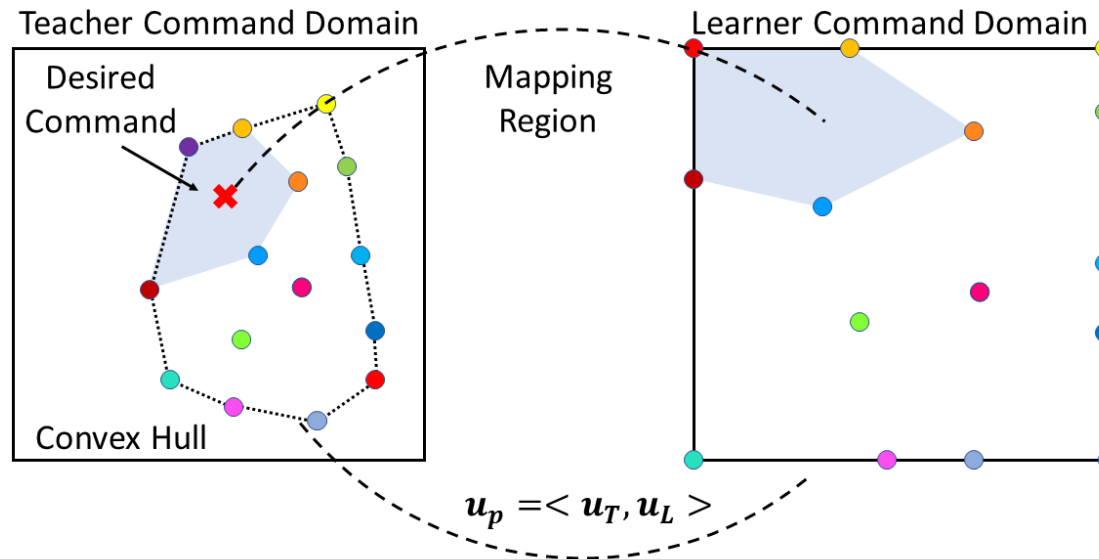
# Methodology: Schwarz-Christoffel Mapping

## Command Pair

- A pair of commands which makes the two vehicles produce the **same motion**.

## Area of Interest

- Given a desired teacher command, the **neighboring commands pairs** construct areas of interest for mapping on both sides.



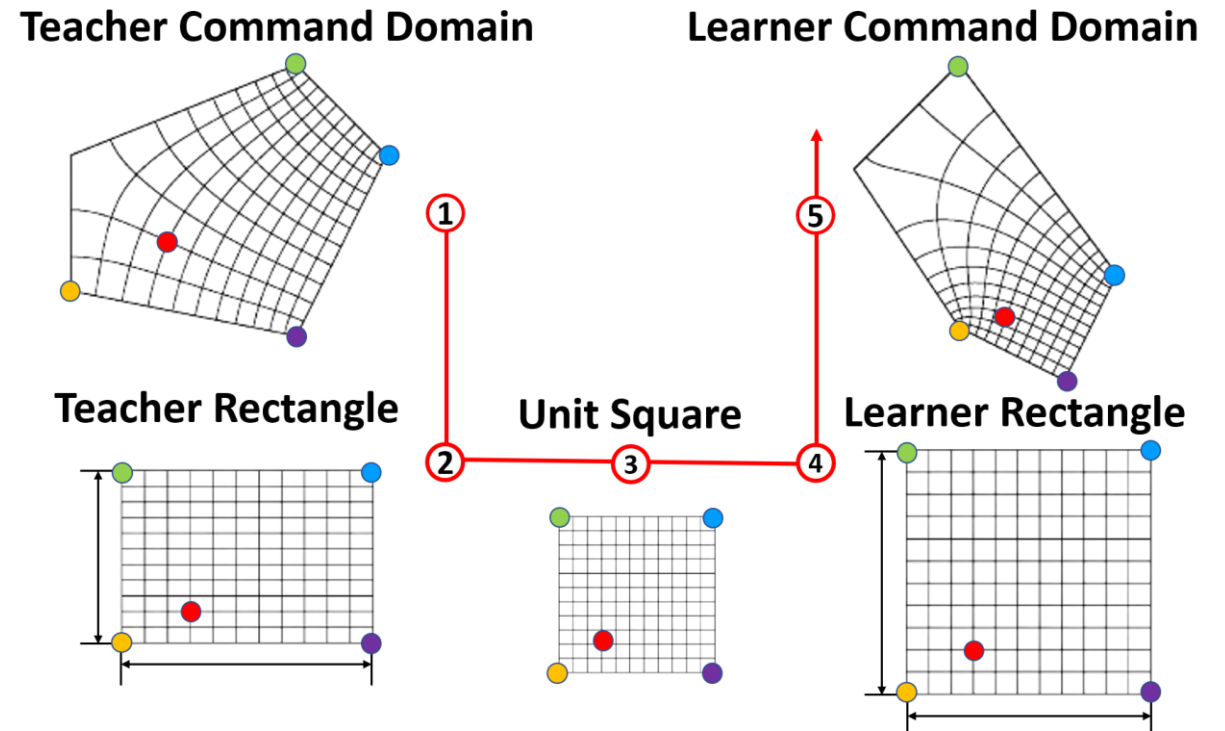
# Methodology: Schwarz-Christoffel Mapping

## Mapping from the teacher command domain to the learner's

- On each side, SCM maps the area of interest to a rectangle of a **unique aspect ratio**.
- A unit square is used to bridge the two rectangles.

## Mapping from a polygon to a rectangle

- Consider the polygon as a generalized quadrilateral.
- Pick 4 vertices as the vertices for the quadrilateral.



# Methodology: Schwarz-Christoffel Mapping

## 1. Mapping from polygon to bi-infinite strip

- Order the polygon vertices  $w_i$  and the pre-vertices  $z_i$  in **CCW**.
- From strip to the generalized quadrilateral.

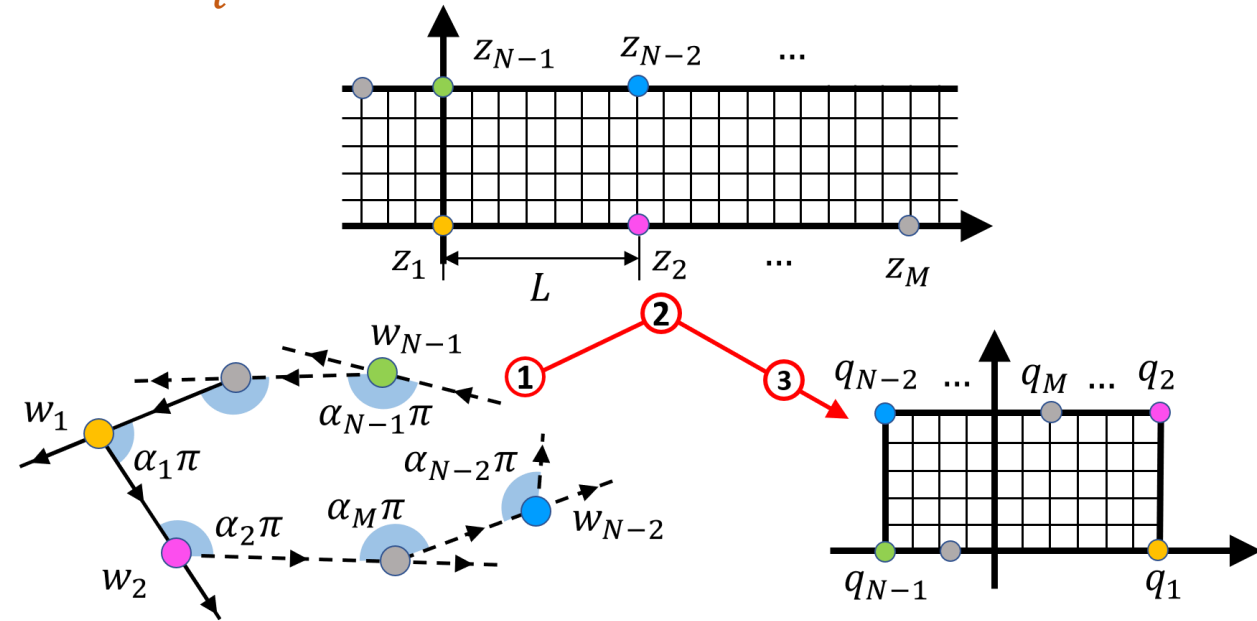
$$w = f_S^\Gamma(z) = A \int_0^z \prod_{j=0}^N f_j(z) dz$$

- The angles  $\alpha_i \pi$  are preserved by

$$f_j(z) = \begin{cases} e^{\frac{1}{2}(\theta_+ - \theta_-)z} & j=0, \\ \{-i \cdot \sinh[\frac{\pi}{2}(z - z_j)]\}^{\alpha_j} & 1 \leq j \leq M, \\ \{-i \cdot \sinh[-\frac{\pi}{2}(z - z_j)]\}^{\alpha_j} & M+1 \leq j \leq N. \end{cases}$$

- Fixing  $z_1 = 0$  to remove constant  $C$ .
- The location of the rest of the pre-vertices  $z_i$

$$\frac{w_{k+1} - w_k}{w_2 - w_1} = \frac{\int_{z_k}^{z_{k+1}} \prod_{j=0}^N f_j(z) dz}{\int_{z_1}^{z_2} \prod_{j=0}^N f_j(z) dz}, \quad k=2, 3, \dots, N-2$$



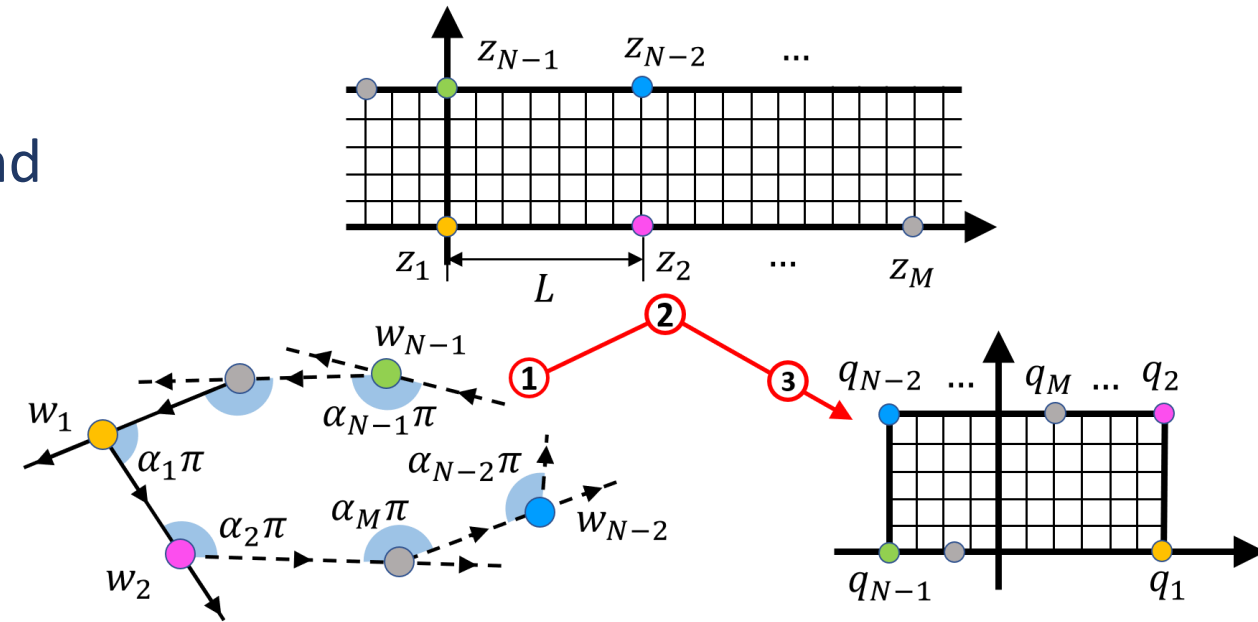
# Methodology: Schwarz-Christoffel Mapping

## 2. Mapping from rectangle to bi-infinite strip

- Leveraging the Jacobi elliptic of the first kind

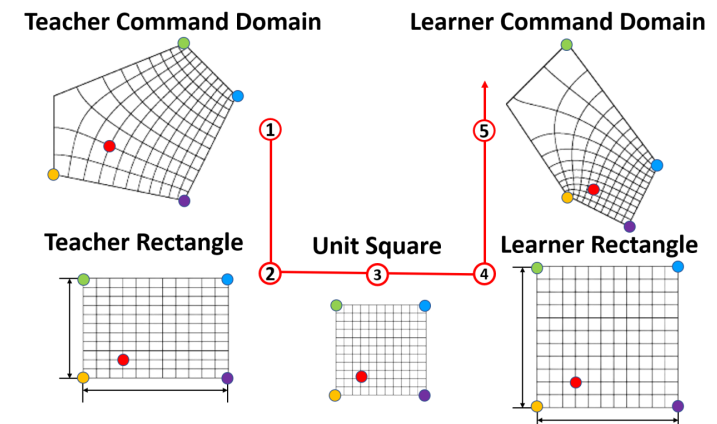
$$z = f_Q^S(q) = \frac{1}{\pi} \cdot \ln(\sin(q|m))$$

- The shape of the rectangle is linked to  $L$



The mapping function from polygon to the rectangle

$$q = f_{SCM}(w) = f_Q^S^{-1}(f_S^\Gamma^{-1}(w))$$

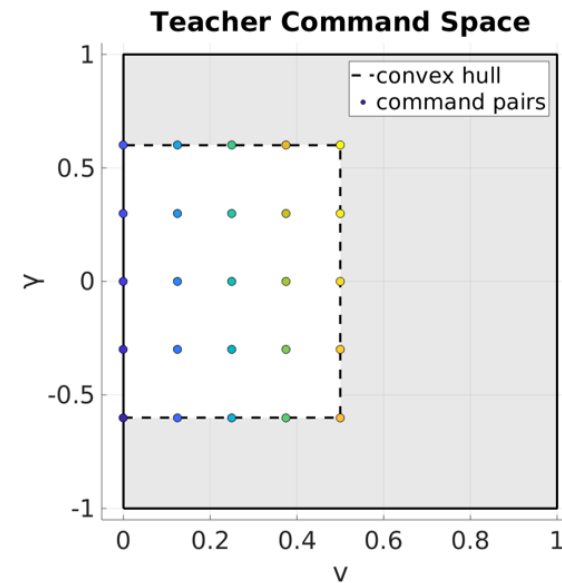
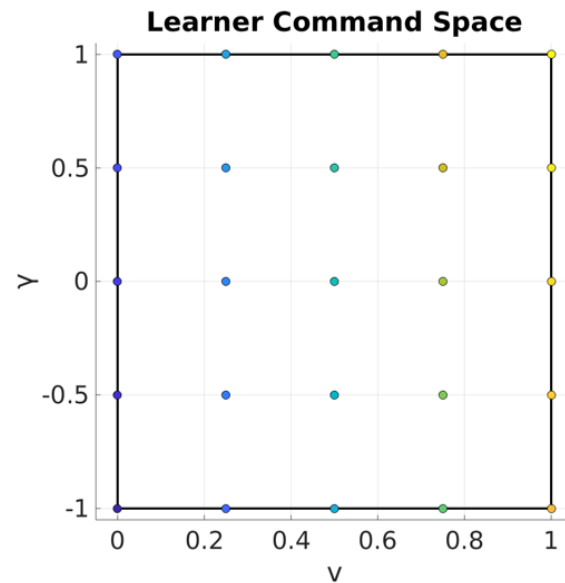




# Methodology: Primitive Path Planning

## Learn the learner's limitations:

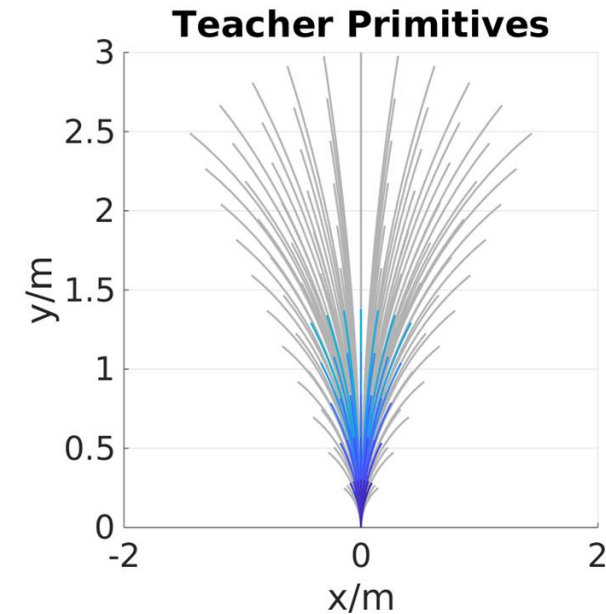
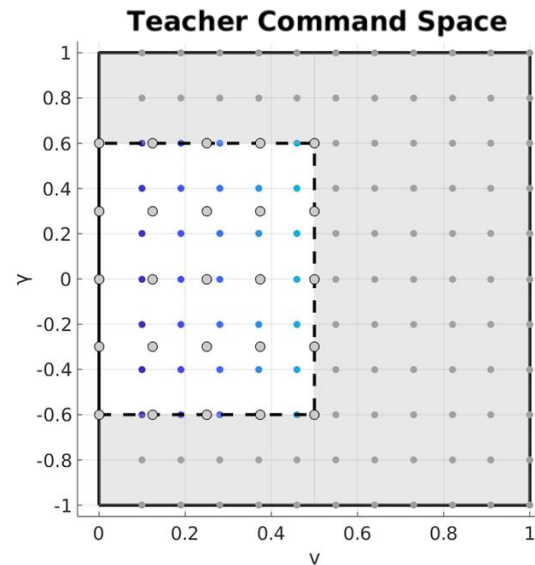
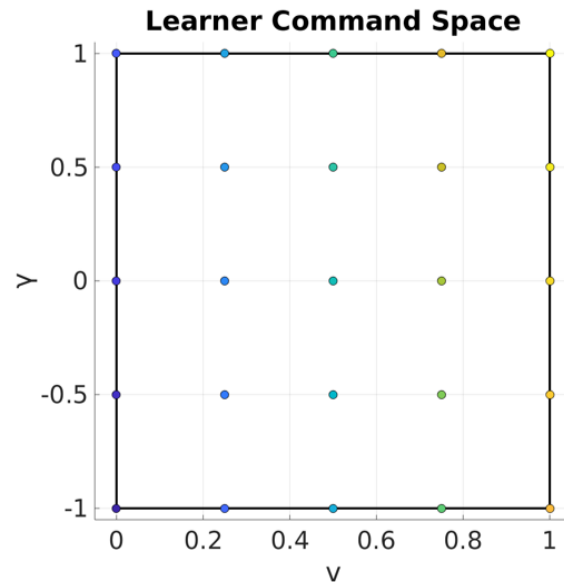
- The learner's **limitation** is characterized on the teacher command space by the **convex hull** built by the **command pairs**.
- The convex hull does not need to cover the whole command space of the learner



# Methodology: Primitive Path Planning

## Motion primitives

- A teacher **motion primitive** is built by feeding the teacher a sequence of the **same command**.
- The primitive whose associated command is **in the convex hull** is used for motion planning.



# Methodology: Primitive Path Planning

## Path planning

- The teacher compares the available **motion primitives** with the **corresponding segment** of the desired path to find the optimal local motion plan.

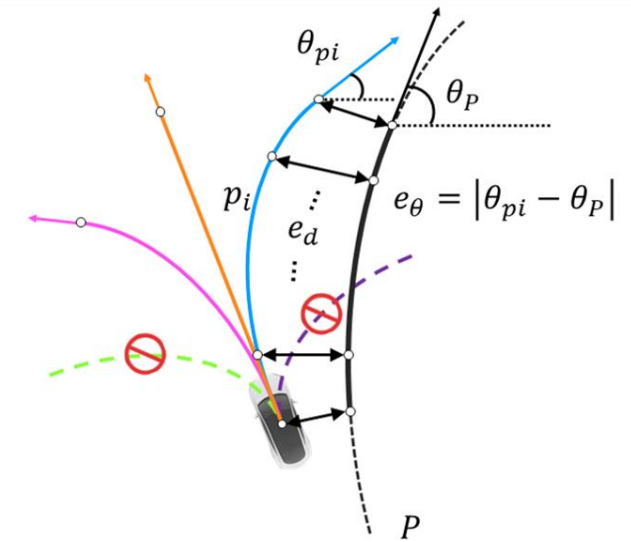
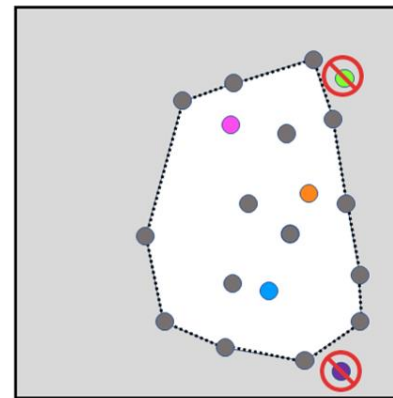
## Evaluate metrics:

- **dynamic time warping (DTW) distance**
- **heading difference**

$$\begin{aligned}\delta_i &= k_d \cdot e_d + k_\theta \cdot e_\theta \\ &= k_d \cdot DTW(P, p_i) + k_\theta \cdot |(\theta_P - \theta_{p_i})|\end{aligned}$$

$$p_i^* = \min_{p_1, \dots, p_i} \delta_i.$$

Teacher Command Domain



An example of picking the optimal local motion

- The associated teacher **command of** the selected **motion primitive** will be the desired command and **mapped to the learner**.

# Methodology: Primitive Path Planning

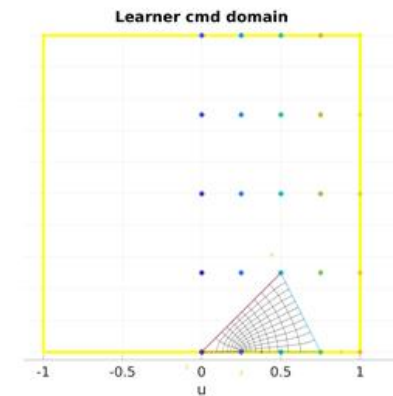
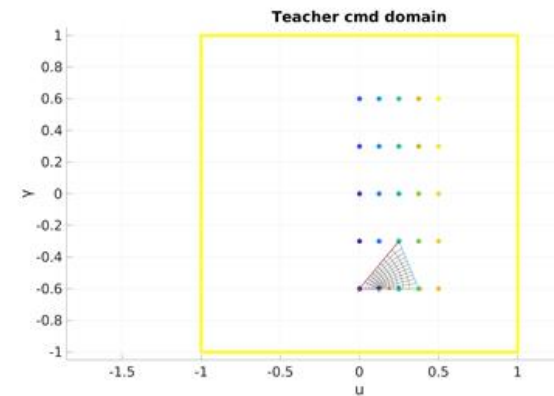
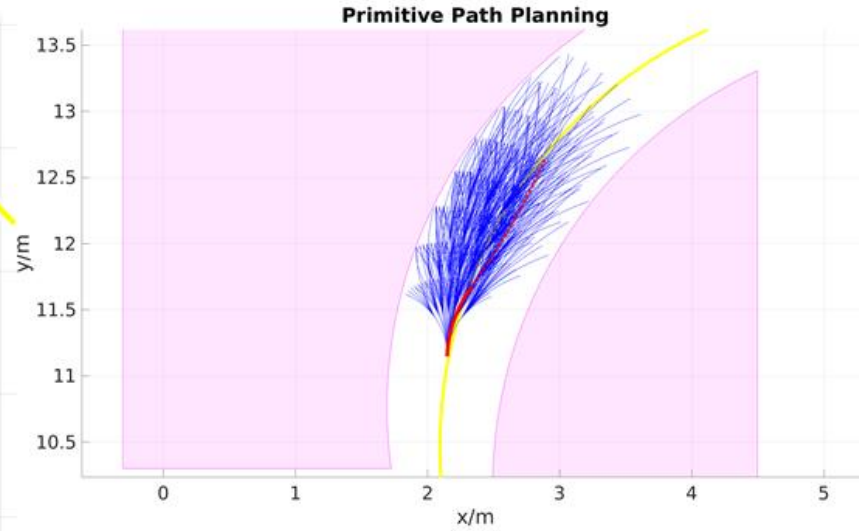
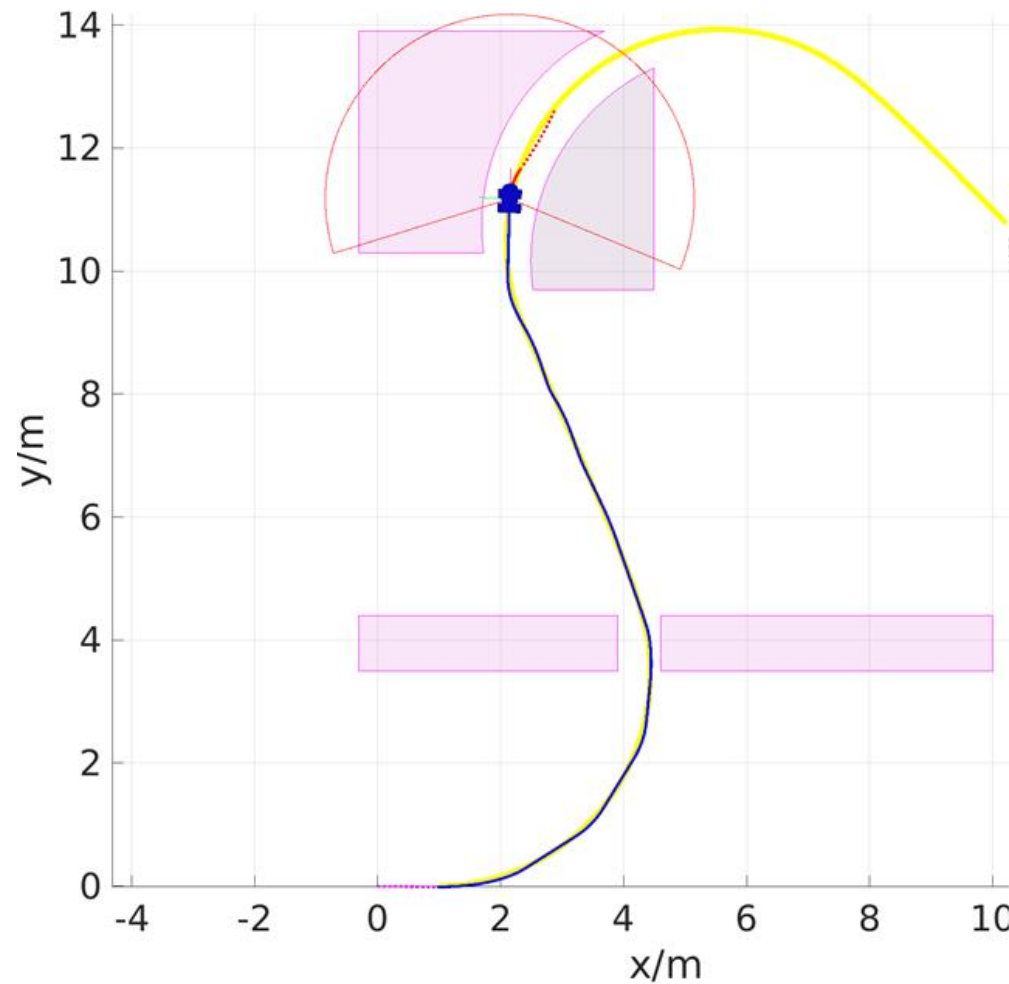
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## Event triggered replanning for safety monitor

- The amount of deviation  $d_{\hat{e}}$  is monitored at runtime
  - In **open area** :  
The deviation is allowed to be **large**.
  - In **cluttered** environment :  
Even a **small deviation** should trigger the **replanning** for safety.

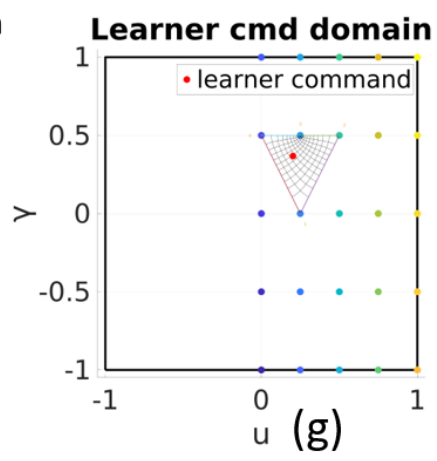
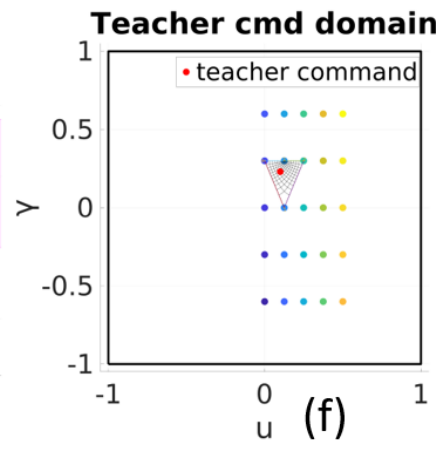
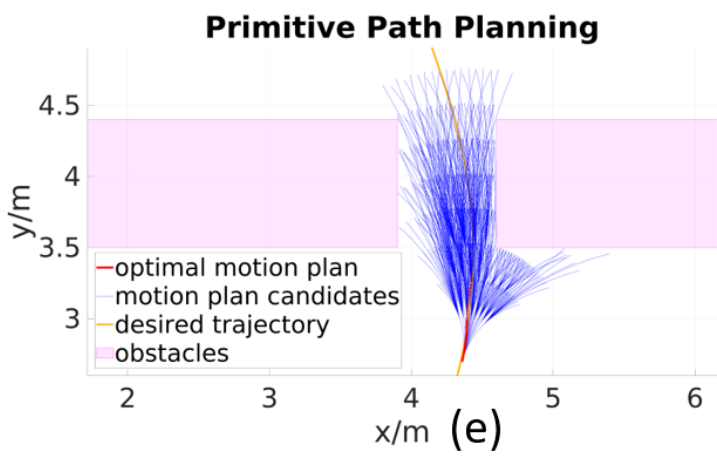
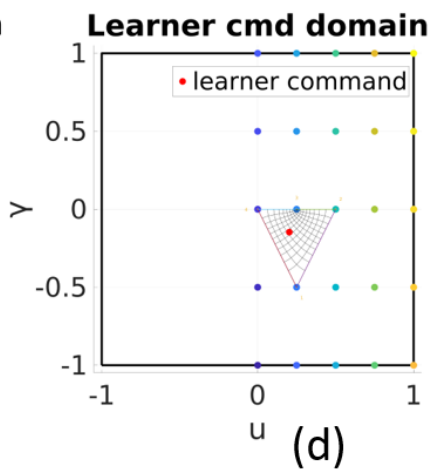
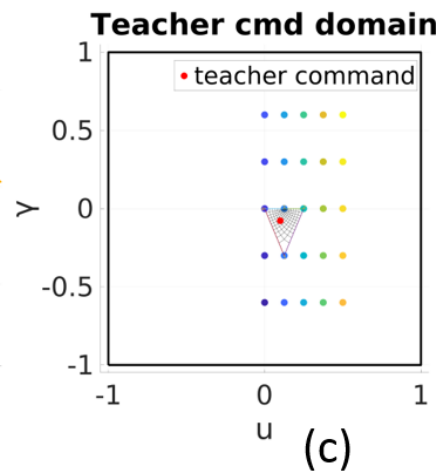
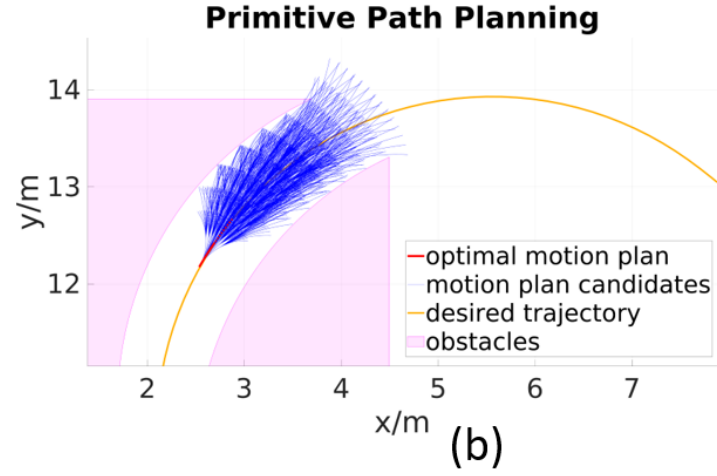
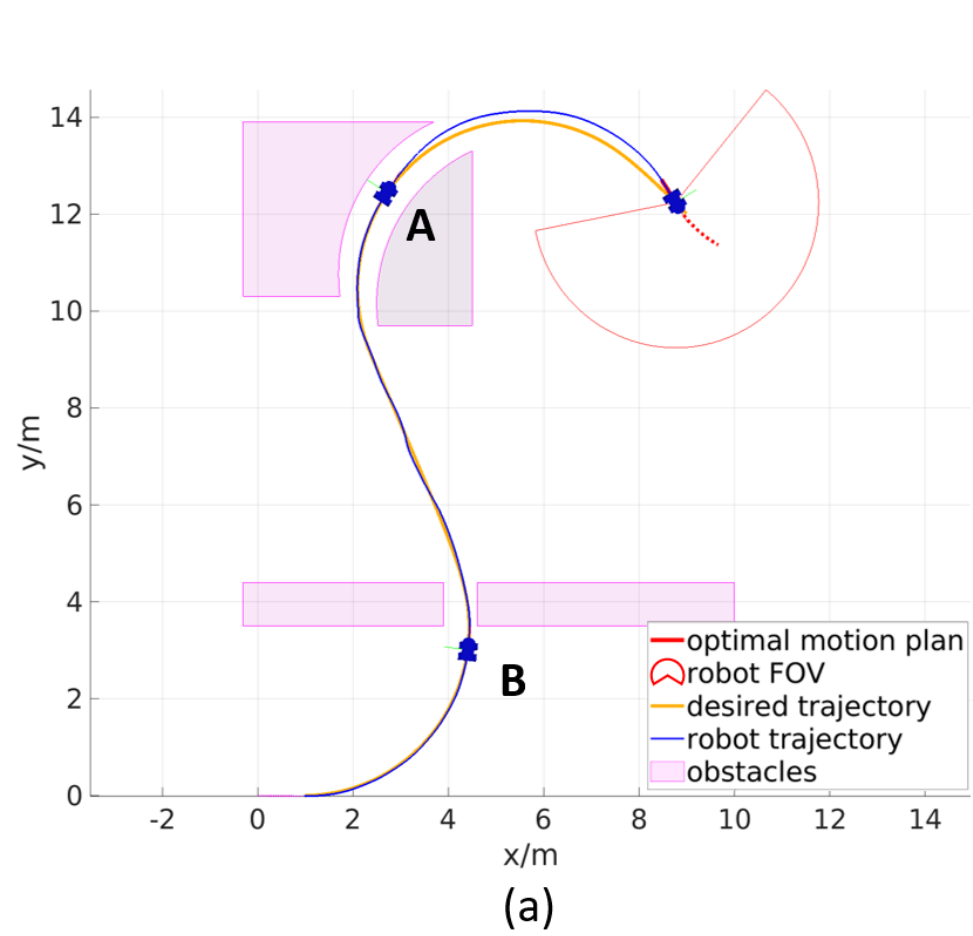
$$\epsilon = \begin{cases} \eta * \min(\|p - o_i\|) & i = 1, 2, \dots, N_o \\ \infty & i = \emptyset, \end{cases}$$

# Simulation Results 1: Original System to Degraded System



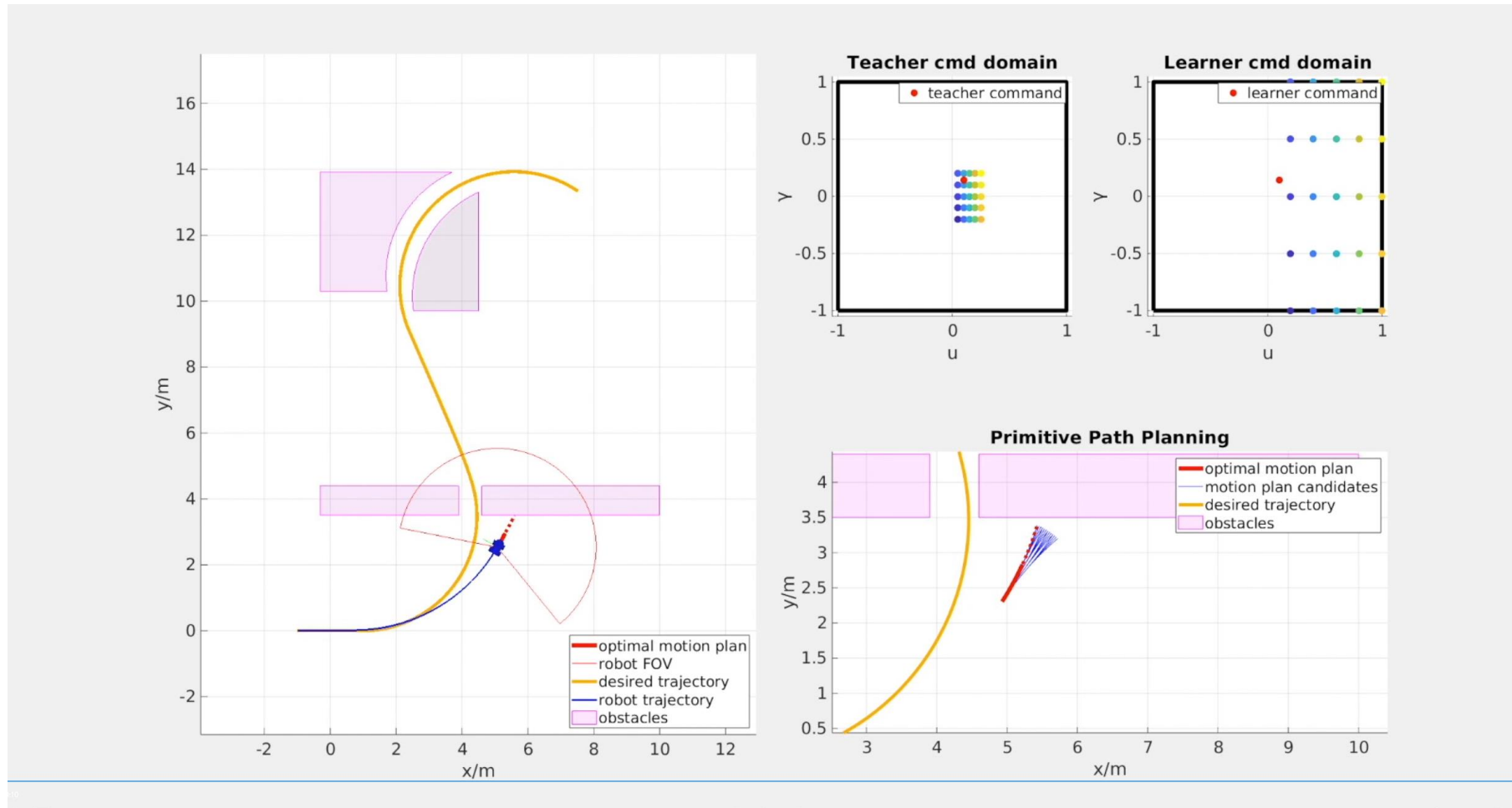
# Simulation Results 1: Original System to Degraded System

- **With** the proposed approach



# Simulation Results 2: Original System to Degraded System

- **Without** the proposed approach



# Experiment Results 1: Simulated UGV to Jackal UGV

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- **With** the proposed approach

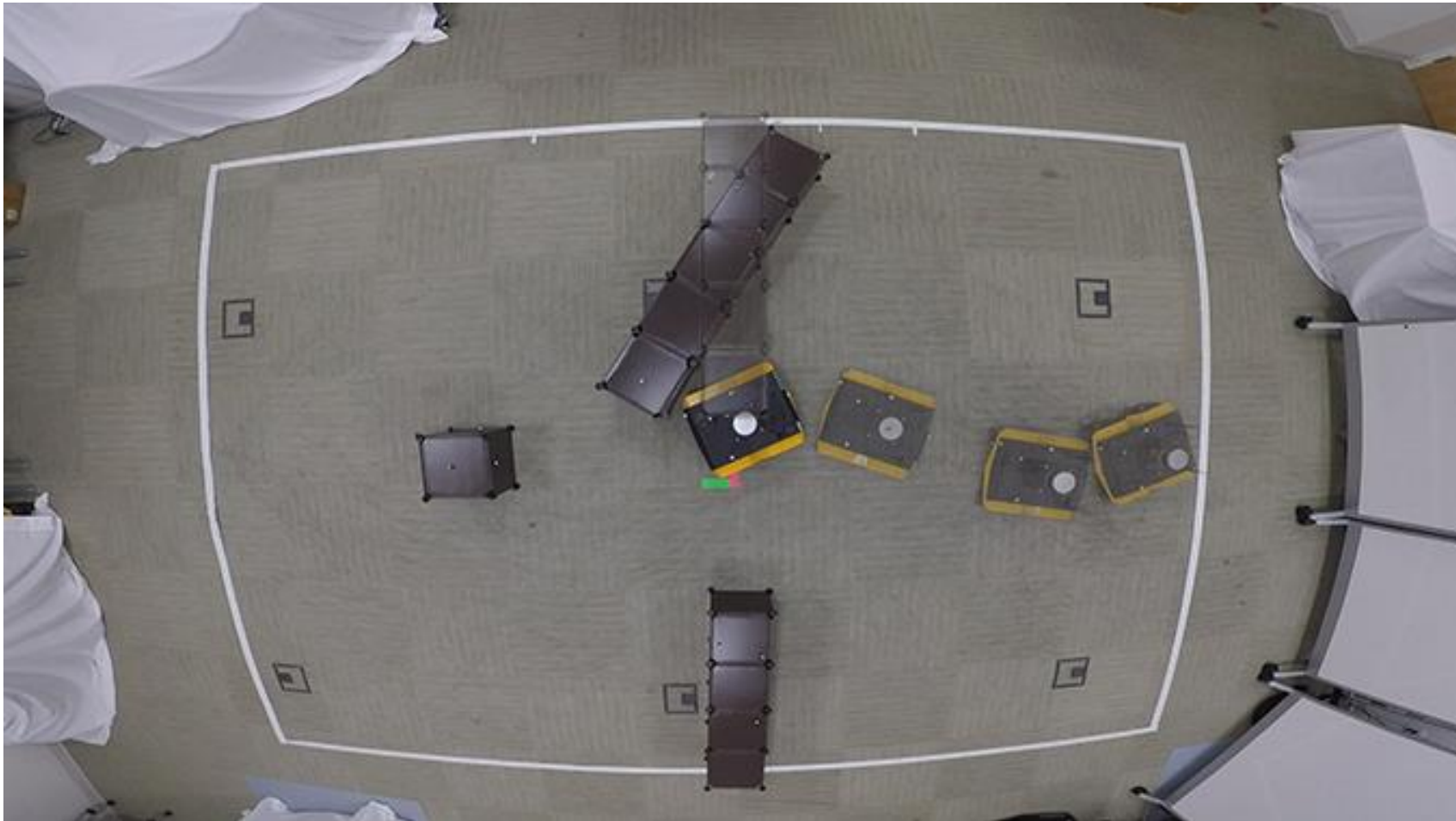




# Experiment Results 2: Simulated UGV to Jackal UGV

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- **Without** the proposed approach



# Experiment Results 3: Simulated UGV to Turtlebot2 UGV

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- **With** the proposed approach



# Conclusions and Future Work

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## Summary:

- A novel *light-weight* transfer learning framework based on conformal mapping.
- Directly maps the control input while the learner's dynamics remain unknown.
- A motion planning policy that adapts to the learner's dynamics.

## Current and Future Work:

- Transfer from a *higher-order* system to a lower-order system.
- Extend our framework to deal with learners with *more capabilities* than the teacher

THANK YOU!

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